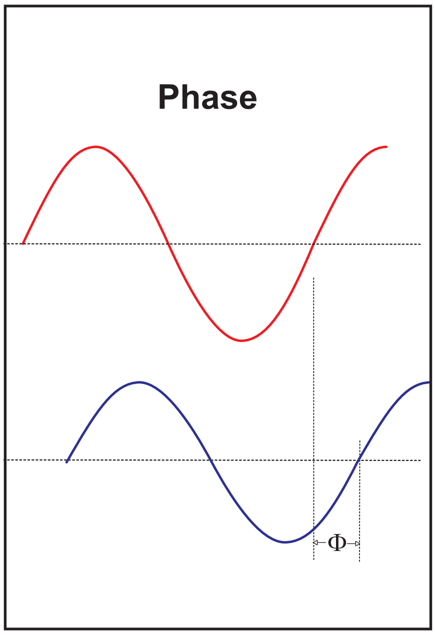
Accurate phase calibration



General

The CNT-81 Timer/Counter Analyzer is an ideal tool for calibrating various time and frequency related parameters, including for example Frequency, Period, Time Interval and Phase.

The very high single shot resolution of 50 ps (1 ps averaged) combined with high-stability time base options, including a Rubidium timebase oscillator, enables fast calibration of time and frequency parameters with very high accuracy, both in field and in the cal. lab.

The built-in statistics functions facilitates calibration, by instant calculation of mean value, standard deviation and max-min peaks over any selected sample size up to 4 billion samples

The included analyzing PC-SW, TimeView™, enables visual feedback and documentation of the measurement process, including measurement vs time graphs, smoothing (digital filtering) and distribution histograms.

Phase measurements

Phase measurements in CNT-81 are combined of two measurements:

1. A time interval measurement (t_{A-B}) between the zero crossings of the signals on input A respectively B.

2. The period (T) of the input A signal

Phase is thereafter calculated as:

$$\Phi = \frac{t_{A-B}}{T} \times 360°$$

There are many factors that affect the uncertainty of the phase measurements. The most important include trigger level timing errors (am I really triggering at the zero-crossing?) and input channel mismatch (are the signal paths, both external and internal, identical?). Both these uncertainty factors are of systematic nature. This application note will describe a procedure to compensate for these potential error sources, and to reduce the uncertainty to below 0.1°.

Practical hints

- An unknown hysteresis is not good when you want to trigger exactly at the zero-crossing. Perform internal hysteresis calibration before the phase measurement (in the AUX menu of CNT-81)
- Select Single measurements combined with statistic calculation of mean value, to improve resolution.
- For frequencies below 500 Hz, use DC-coupling instead of AC-coupling to eliminate phase error due to tolerances in AC-coupling capacitors.
- Optimum input voltage range for highest accuracy is between 0.3 and 3 Vrms (3 and 30 Vrms with 10x input attenuation). Higher input voltages than 30 Vrms need external attenuation. Lower input voltages than 0.3 Vrms may cause increased uncertainty.
- For signals that contain HF-noise, use two external matched low pass filters.
- To reduce the uncertainty due to trigger level offsets and inequality of input channels (including external cabling and filters), swap externally the start and stop signals and take the average value of these measurements.
  If the first reading is Φ1 and the second reading (with swapped input signals) is Φ2, then the most accurate estimate of the "true" phase angle is:

$$\Phi = \frac{\Phi 1 + (360° - \Phi 2)}{2}$$

Procedure for Phase calibration of a phase generator

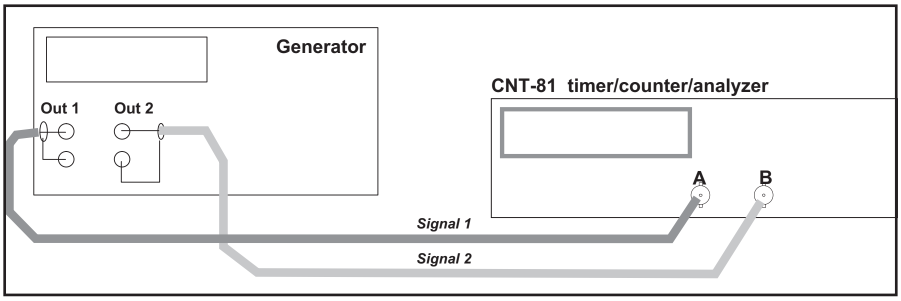


Fig. 1 *Setting up the CNT-81 for phase measurements*

Before performing a calibration, be sure that both the generator and the CNT-81 are in a controlled climatic environment ant that both instruments are warmed up according to the manufacturers specification.

A. Set-up the CNT-81

- Select function PHASE A-B
- Set input channels A and B to:
  - DC-coupling
  - correct termination (50Ω or 1 MΩ)
  - positive slope
  - trigger level 0 V
  - AUTO trigger Off
- Select SINGLE measurements
- Set measurement time to 100 μs (to speed up the readings)
- Select Statistics and MEAN with 100 samples. (To reduce jitter and improve resolution)

B. Make an internal hysteresis calibration:

- Disconnect any signal cable from the input channels A and B on CNT-81
- Push button AUX MENU, select CAL HYST and push ENTER.
- Reconnect signal cables.

C. Connect the Generator to be calibrated

Connect the two phase-shifted outputs from the generator to Channel A and B of CNT-81. If the output signals contain HF-noise and spurious signals, then use two matched LP-filters (3 dB bandwidth at least 25 times above max. signal frequency) to reduce trigger error. See fig. 2. Many common types of generators and LF-voltage calibrators, produce high-frequency noise and require the use of LP-filters. Note that unmatched filters may introduce an extra phase shift, but this error is fully compensated for by swapping the signals (incl. filters) according to point E below.

D. Set-up for Voltage to Current Phase measurements:

Some generators do generate a phase shift between a Voltage output and a separate Current output. The CNT-81 has no current input and thus the current output of the generator must be converted to a voltage, via a shunt-resistor with a suitable resistance and power rating. See figure 3.

For minimum uncertainty, select a resistance value to reach approx. the same input voltage levels for both channels. E.g. for a Voltage output of 1 V rms and a current output of 300 mA rms, select a shunt resistor value of 3.3 Ω (½ W).

E. Repeat measurement with swapped inputs:

Phase measurements in CNT-81 have a typical basic uncertainty of some tenths of a degree; the actual value depends on the signal properties.

For uncertainty levels below 0.5° two series of measurements must be made. First with normal connections and thereafter with the input signals interchanged. See figure 4.

- First signal 1 is connected to input A and signal 2 to input B, giving the value $\phi 1$.
- Secondly signal 1 is connected to input B and signal 2 to input A, giving $\phi 2$.
- Finally the average value of the results $\phi 1$ and $\phi 2$ is calculated:

$$\phi = \frac{\phi 1 + (360° - \phi 2)}{2}$$

This procedure will compensate for systematic uncertainties due to trigger level errors and other systematic channel differences..

Note 1: It is important to swap the complete external signal path, also possible LP-filters.

Note 2: Do *not* use the internal SWAP A⇔B button)

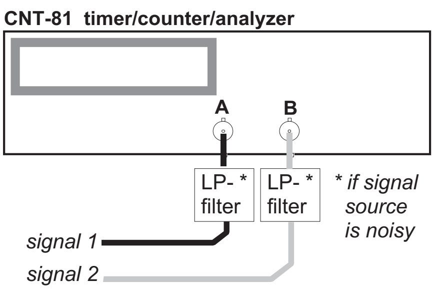


Fig. 2 *Use external matched LP-filters if the signals are noisy*

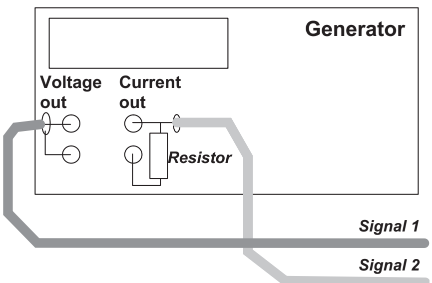


Fig. 3 *Converting current to voltage in voltage-to-current phase measurements*

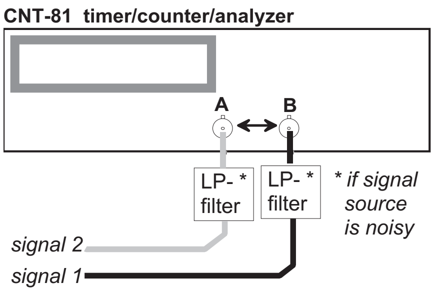


Fig. 4 *Swap the external signal connections*

Appendix

Calculations acc. to GUM (Guide to the expression of Uncertainty in Measurements by BIPM a.o.)

General:

1. Identify all contributing uncertainty factors

2. Express the influence of all factors (whether random or systematic) on the measured value as one std deviation. Normally a systematic uncertainty is given as limit values ±*a*. To calculate the standard deviation requires knowledge of the distribution. When in doubt (which is the normal case) it is safe to assume a rectangular distribution, for which the std deviation is $a/\sqrt{3}$

3. If all factors are un-correlated, express the Combined Uncertainty (*u*) as the root of the sum of the squares of all contributing uncertainty factors (u1,u2, ..):

$$u = \sqrt{u_1^2 + u_2^2 + \ldots}$$

4. To obtain a higher confidence level, multiply *u* by 2 to get Expanded Uncertainty on a "2σ"-level (*U*):

$$U = 2 \cdot u \qquad (k=2)$$

Phase measurements:

Phase (A-B) between the input signals on inputs A and B is measured as two consecutive measurements:

- Period (*T*) of signal on input A

- Time Interval (TI_{A-B}) from set trigger levels (normally zero crossings) of input A to input B

Phase is thereafter calculated as:

$$\Phi = \frac{TI_{A-B}}{T} \times 360^\circ$$

Phase is an indirect measurement, based on two independent time measurements, the Period and the Time Interval. The combined uncertainty of the phase is called $u(\Phi)$ and the combined uncertainties of the period and time interval measurements are called *u(T)* respectively *u(TI).*

There are many uncertainty factors, affecting both Period and Time Interval. Some are neglectable and some are dominant. We will examine in depth the contributing factors shortly.

Lets first exclude one possible uncertainty sources, i.e. the timebase uncertainty. In the phase measurement, it does not matter at all if the oscillator frequency is correct or not, since a change in oscillator frequency will affect both the Period and the Time Interval measurement proportionally, on each side of the division line.

Lets make a more mathematical approach. The phase ϕ is a function of the time interval and the period, and can be expressed as $\phi = \phi(TI, T)$.

If we assume that the period and time interval measurements are un-correlated, the uncertainty of the measurement can be expressed as:

$$\left(\frac{u(\Phi)}{\Phi}\right)^2 = \left(\frac{u(T)}{T}\right)^2 + \left(\frac{u(TI)}{TI}\right)^2$$

We will show that the Period uncertainty can be neglected, because all remaining uncertainty factors are of a random nature, and can be averaged to a neglectable uncertainty. It is Time Interval that is critical, because - unlike Period - its uncertainty is dependent on the *systematic* difference between the 2 input channels.

When is a contributing uncertainty element neglectable? A suitable criteria could be when its contribution to uncertainty is below the display resolution threshold of 0.01°. This corresponds to a relative uncertainty of $< \frac{0.01^\circ}{360^\circ} \approx 3 \cdot 10^{-5}$

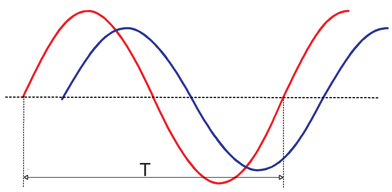
Signal Example:

Let us analyze the uncertainty of a Phase measurement between zero crossings of two sine-wave signals with:

Signal/Noise-ratio (SNR) = 60 dB
Frequency = 1 kHz (Period = 1ms)
U1 = 2 Vrms
U2 = 1 Vrms.

The mean value of 100 samples is taken via the Statistics function in CNT-81.

A: The uncertainty of the period measurement

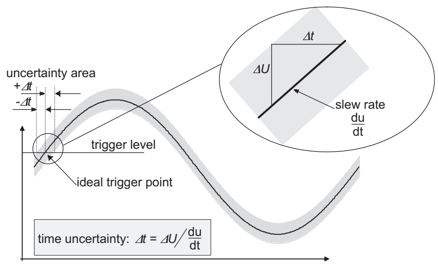


The only systematic uncertainty factor is the timebase uncertainty, which has no influence on the phase measurement. Therefore only random uncertainty factors are considered.

Random uncertainty factors:

1. Resolution (single-shot):
u1 = 50 ps (rms)
(reduced to 5 ps, via averaging)

2. Start trigger point uncertainty due to noise:



$$u2 = \frac{\sqrt{(\text{internal noise})^2 + (\text{external noise})^2}}{\text{Signal slew rate at trigger point}}$$

For small values of ΔU the signal slew rate is approx. equal to the slew rate at the zero crossing.

For a sine-wave signal *u(t)=û· sin(ωt)*, the slew rate $\left(\frac{du}{dt}\right)$ at the zero-crossing equals: $\frac{du}{dt} = \hat{u} \cdot \omega \cdot \cos(0) = 2\pi f \cdot U_{rms} \cdot \sqrt{2}$

The internal noise in CNT-81 is 100 μVrms, which gives us:

$$u2 = \frac{\sqrt{(100\mu V)^2 + (ext.noise)^2}}{2\pi \cdot freq \cdot U_{rms} \cdot \sqrt{2}} =$$

$$= \sqrt{\left(\frac{100\mu V}{U_{rms}}\right)^2 + \left(\frac{1}{SNR}\right)^2} \times \frac{1}{2\pi \cdot freq \cdot U_{rms} \cdot \sqrt{2}} =$$

$$= \sqrt{0.00005^2 + 0.001^2} \times (2\pi \cdot 1000 \cdot 2 \cdot \sqrt{2})^{-1} \approx$$

$$\approx 56ns\ (\text{rms value})$$

(reduced to approx. 6 ns, via averaging)

3. Stop trigger point uncertainty due to noise:

This is exactly the same as the start trigger point uncertainty, 6 ns after averaging. Start and stop of the period occurs at the same trigger point.

4. Combined uncertainty *u(T)* for period:

$$u(T) = \sqrt{u1^2 + u2^2 + u3^2} =$$

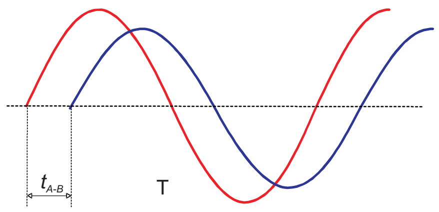
$$= \sqrt{0^2 + 5.6^2 + 5.6^2} \approx 7.9ns\ (rms)$$

The relative uncertainty is

$$\frac{u(T)}{T} = \frac{7.9ns}{1ms} \approx 8 \cdot 10^{-6}$$

and the Period measurement contributes with an neglectable phase uncertainty of 0.003° ($8 \cdot 10^{-6} \times 360^\circ$)

B: The uncertainty of the time interval measurement



Random uncertainty factors:

1. Resolution (single-shot):

u1 = 50 ps (rms)

(reduced to 5 ps, via averaging)

2. Start trigger point uncertainty due to noise is the same as for Period:

$$u2 = \frac{\sqrt{(\text{int.}\,noise)^2 + (ext.\,noise)^2}}{\text{Slew rate at trigger point}} \approx 56ns$$

(reduced to 5.6 ns, via averaging)

3. Stop trigger point uncertainty due to noise is higher, because the slew rate is lower, due to lower amplitude of the top signal (1V vs. 2V rms):

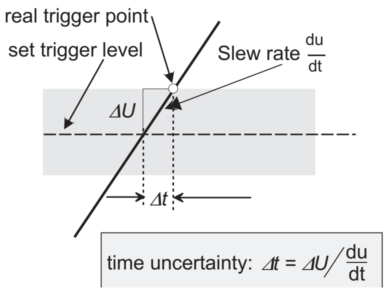
$$u3 = \frac{\sqrt{(\text{int.}\,noise)^2 + (ext.\,noise)^2}}{\text{Slew rate at trigger point}} \approx 0.11\mu s$$

(reduced to 11 ns, via averaging).

Systematic uncertainty factors:

The following systematic uncertainty factors can not be reduced via statistical averaging:

4. Start trigger point uncertainty due to trigger level offset (U_A) in the start channel.



For CNT-81, this offset from zero is less than 2.5 mV after an internal hysteresis calibration:

$$\Delta t = \frac{U_A}{\text{slew rate at trigger point}} =$$

$$= \frac{25mV}{2\pi \cdot freq \cdot U1_{rms} \cdot \sqrt{2}} =$$

$$= \frac{25mV}{2\pi \cdot 1000s^{-1} \cdot 2V \cdot \sqrt{2}} \approx 0.14\mu s\ (\text{limit value})$$

This is a limit value, which is assumed to have a rectangular distribution. You get the corresponding standard uncertainty by dividing with $\sqrt{3}$, which gives:

$$u4 = 0.14/\sqrt{3}\mu s \approx 80ns$$

5. Stop trigger point uncertainty due to trigger level offset (U_B) in the stop channel. Also here, the offset from zero is less than 2.5 mV after an internal hysteresis calibration. The uncertainty for the stop trigger point is twice as large as for the start trigger point, due to the lower slew rate of the stop signal:

$$u5 = 2 \cdot u4 \approx 0.16\mu s$$

6. Channel asymmetry uncertainty

The internal time difference from the input BNC:s to the measurement kernel is less than 500 ps.

$$u6 = 500/\sqrt{3} \approx 300ps$$

Combined uncertainty u(TI) for time interval:

We have identified in total 6 uncertainty factors for the time interval measurement. See below, where also the uncertainty contribution to the Phase is listed:

u1 = 5 ps or 0.000002°

u2 = 5.6 ns or 0.002°

u3 = 11 ns or 0.004°

u4 = 0.08 μs or 0.03°

u5 = 0.16 μs or 0.06°

u6 = 0.3 ns or 0.0001°

Since the combined uncertainty of the *period* measurement contributed to Phase uncertainty with only 0.003°, we can draw the following conclusion:

The by far dominant uncertainty factor in the phase measurement, is the systematic uncertainty of the time interval measurement, due to the zero-level trigger offset (u4 and u5).

For a given phase measurement, the period can be regarded as constant and invariable and all uncertainty is associated with the Time Interval measurement. Thus the uncertainty $u(\phi)$ can be expressed as:

$$u(\phi) = \frac{u(TI)}{T} \times 360^\circ$$

This leads to an Expanded Uncertainty *U* (*k*=2) in this example of:

$$U(\phi) = 2x\frac{u(TI)}{T} \times 360^\circ =$$

$$= 2x\frac{\sqrt{u1^2 + u2^2 + u3^2 + u4^2 + u5^2 + u6^2}}{T} \times 360^\circ =$$

$$= 2x\frac{\sqrt{0^2 + 0^2 + 0^2 + 0.08^2 + 0.16^2 + 0^2}}{1ms} \times 360^\circ =$$

$$= \frac{0.36\mu s}{1ms} \times 0.13^\circ$$

C. Reducing the systematic uncertainty, by swapping the inputs

As mentioned above, uncertainty levels significantly below 0.5° is difficult to reach in a standard measurement without special care. A procedure, where the result is obtained from two consecutive measurements, with swapped input signals in the second measurement, will significantly reduce the uncertainty.

The start-stop trigger timing uncertainties are shown (greatly exaggerated) in the figure below.

The two input signals have a fixed phase relation of Φ1 (from u1(t) to u2(t)), and Φ2 (from u2(t) to u1(t). These are the "true" values, where Φ2=360°-Φ1.

The actually measured values are $\varphi 1$ between trigger point 1 and 2 in measurement number 1 and $\varphi 2$ between trigger points 3 and 4 in measurement number 2.

The trigger level is set to 0 V and a hysteresis calibration has been made.

We ignore all other uncertainties, than those due to systematic trigger level offsets. We further assume that these offsets (UA and UB) are fixed and stable for the CNT-81 during the complete measurement sequence. This means that the correlation coefficient ρ=+1 for the trigger point uncertainties in the respective measurements.

We have *identical* systematic uncertainty $u(\varphi)$ in both measurements $\varphi 1$ and $\varphi 2$. The calculated value from the two measurements is :

$$\varphi = \frac{\varphi 1 + 360^\circ - \varphi 2}{2} = \frac{\varphi 1 - \varphi 2}{2} + 180^\circ$$

According to uncertainty calculation theories, the combined uncertainty u_C of the expression is then:

$$u_c = \frac{u(\varphi 1) - u(\varphi 2)}{2} = 0$$

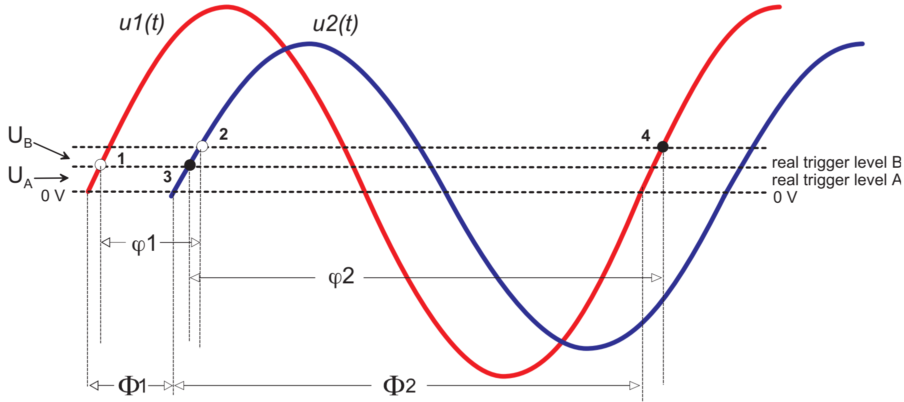
and not:

$$u_c = \frac{\sqrt{u(\varphi 1)^2 + u(\varphi 2)^2}}{2}$$

as is the case with uncorrelated variables (ρ=0).

We have thus eliminated the systematic trigger level uncertainty, and need only to consider the other, less dominant factors.

In the example above, with systematic uncertainties *u4* and *u5* eliminated, it is only *u2* and *u3* (start and stop trigger error due to noise) that contribute, which gives us the residual combined uncertainty in the Time Interval measurement:



○ Trigger points in first phase A-B measurement (from signal u1 to signal u2)
● Trigger points in second phase A-B measurement (from signal u2 to signal u1 after swapping inputs)

$$u(TI) = \frac{\sqrt{u2^2 + u3^2 + u2^2 + u3^2}}{2} = $$

$$= \frac{\sqrt{2 \cdot 6^2 + 2 \cdot 11^2}}{2} \approx 9ns$$

contributing to a phase uncertainty of $\frac{9ns}{1ms} \times 360^\circ \approx 0.003^\circ$

Since the systematic uncertainty is eliminated, we need also to take into account the uncertainty of the period measurement, which we have neglected so far. From the text above in the example, we saw that the period measurement also contributed with an uncertainty of 0.003°. By combining both measurements (the "original" and the swapped) the relative period uncertainty is reduced with $\sqrt{2}$ to approx. 0.002°

Finally we get the Expanded Uncertainty (k=2) by combining the relative uncertainties of the period and the time interval measurements:

$$U(\phi) = 2 \times \sqrt{0.002^2 + 0.003^2} \approx 0.007^\circ$$

This is a tremendous reduction of the 0.13° uncertainty in the original measurement only.

Although the trigger point uncertainty is the major source of (systematic) uncertainty, it is recommended to make a complete uncertainty calculation based also on other sources of uncertainty (like random variations due to the SNR and resolution and systematic channel delay mismatch). See the operators manual for CNT-81.

Please remember that uncertainties caused by external and internal noise can be reduced by statistical averaging to a level far below the uncertainty caused by the trigger level offset.

Summary

The CNT-81 is an excellent tool for making high-accuracy phase calibration. Its high time and trigger level resolution, low channel mismatch plus the unique internal hysteresis calibration, build the foundation for reliable measurements.

You get accurate results from direct measurements, but when you also combine it with a repeated measurement with swapped inputs, you get a remarkably low phase uncertainty.

It normal cases, it is the time interval, and not the period measurement that sets the accuracy limit.

Please note that the individual uncertainty factors contribute more or less to the combined uncertainty depending on the properties of the signal:

- *Resolution* (50 ps) is normally neglectable, except for extremely high input signal frequencies (>30 MHz). But remember to use statistical averaging to improve resolution
- *Systematic channel mismatch* (300 ps) is neglectable for low frequency signals, but can be quite significant for HF-signals. However this systematic error is also eliminated by swapping the input

Trigger point uncertainty is the dominant factor for all low slew rate signals. For square wave signals however, even this factor becomes neglectable